Maximizing Social Welfare in Fractional Hedonic Games using Shapley Value

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***Abstract*—Fractional hedonic games (FHGs) are extensively studied in game theory and explain the formation of coalitions among individuals in a group. This paper investigates the coalition generation problem, namely, finding a coalition structure whose social welfare, i.e., the sum of the players' payoffs, is maximized. We focus on agent-based methods which set the decision rules for each player in the game. Through repeated interactions the players arrive at a coalition structure. In particular, we propose CFSV, namely, coalition formation with Shapley value-based welfare distribution scheme. To evaluate CFSV, we theoretically demonstrate that this algorithm achieves optimal coalition structure over certain standard graph classes and empirically compare the algorithm against other existing benchmarks on real-world and synthetic graphs. The results show that CFSV is able to achieve superior performance.**

***Index Terms*—fractional hedonic games, coalition formation, Shapley value**

I. Introduction

The formation of coalitions is a common phenomenon in human society when individuals or groups try to attain greater mutual benefits by establishing alliances. A simple example is when students in the same laboratory form study groups through sharing resources about a research topic. Another example is when companies establish inter-organizational agreements to gain market dominance. *Cooperative games* are natural models that has been used to model the dynamics of coalition formation. In a cooperative game, players can freely form *coalitions* in which they cooperate. A *coalition structure* is a partition of the players into disjoint groups.

It is common knowledge that the formation of a coalition structure is largely driven by individualistic preferences and such preferences are affected by the social relationships among the individuals. For example, a politician would prefer to join a political fraction which has a higher proportion of like-minded members who share the similar values [1]. In a social network, a person will be more likely to participate in a group activity if the group offers her higher popularity [2], [3]. *Fractional Hedonic Games* (FHGs), first proposed by Aziz, Brandl, and Brandt, is a model that captures the payoff of a player in such a scenario [1]. In this model, the players' preferences regarding which coalitions to join rely solely on social ties among individuals who are members of the respective coalitions. In this sense, a FHG is a cooperative game that is defined over a graph structure, i.e., the social network formed by the players of the game and their social connections.

There have been many works that explore social connections from a game-theoretic perspective [4]–[6]. It is widely known that a social network exhibits some specific structural traits such as high clustering coefficient and low average path length which characterize the so-called small-world property. An established line of research aims to explore the dynamics behind small-world networks [7]–[9]. One important question in analyzing small-world network structures is to explore how nodes in the network, being treated as "autonomous agents", utilize their social link to form reasonable coalitions. Such coalition structure should reveal insights such as community structure [10], [11] and how information may effectively diffuse through the network [12], [13].

One crucial question along this line of investigation is the so-called *coalition generation problem*, which takes as input a cooperative game and looks for a coalition structure with the maximum social welfare, i.e., the sum of utilities of all coalitions. To solve this problem, one normally adopts a top-down approach and develops centralized optimization techniques to find the desired coalition structure. This approach, however, does not utilize the autonomy of each player. A different computational paradigm, on the other hand, views each player as a separate computation unit and achieves a coalition structure through simulating the decision making and coordination process of the crowd. The advantage of this paradigm is that by delegating the computation to individuals, the overall system is scalable and adaptable to changes.

Contribution: This paper focuses on the agent-based paradigm for solving the coalition generation problem. The benchmark approach simulates coalition formation by giving each individual player a payoff that is defined by FHG formulation, namely, the payoff given to an individual equals the proportion of adjacent nodes in the coalition verses the size of the coalition. An individual may interactively evaluate the possible payoffs if she joins any coalition and chooses the coalition that will provide maximum payoff. We adopt a

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similar framework but with a twist: We use the *Shapley value* [14] as our welfare distribution method instead of the one prescribed by FHG. Since the individual in this formulation will get a different reward than the in FHG, our mechanism ends up operating on a new game-theoretic model, where the utility of each coalition is the same as in FHG, but players obtain a payoff that is proportional to their Shapley value in the coalition. This will have an impact on the resulting coalition structure without changing the definition of the social welfare. The main contributions of this paper are as follows: (1) We define the schemes of *coalition formation with popularity-based welfare distribution* (CFPO) and *coalition formation with Shapley value-based welfare distribution* (CFSV) in the hope to identify socially optimal coalitions in FHGs. CFPO will be used as a benchmark to compare with CFSV. (2) We theoretically prove the optimality of CFSV over some standard graph classes. (3) We propose an approximate algorithm CFSV_8 to efficiently compute the CFSV especially for larger graphs. (4) We experimentally analyse the social welfare of coalition structures on both real-world and synthetic networks and demonstrate the superiority of CFSV over other methods.

Related work: Aziz et al. proved that the computational complexity of maximizing social welfare is NP-hard in a simple symmetric FHG and proposed approximation algorithms which searches for maximal matchings [15]. However, these algorithms can only form 2-player coalitions when there is a matching and 1-player coalitions for unmatched single nodes. The algorithm is therefore limited as the maximum number of players in a coalition is 2. Flammini et al. investigated forming coalitions in FHGs online, where a new player appears at each time-step and decides whether to join an existing coalition or to form a new coalition by himself [16]. Nevertheless, they restricted the number and sizes of coalitions to restrict the possible coalition structures which can be formed. Monaco, Moscardelli and Velaj focused on the egalitarian social welfare (i.e., utility of worst-off agent) when modified FHGs reach strong Nash stability, Nash stability and core stability. They gave bounds of the price of anarchy and price of stability in this situation [17]. Liu and Wei defined a popularity game (i.e., a simple symmetric FHG) where the utility of a player represents her popularity [5]. They discussed whether the grand coalition structure (all players form a coalition) is core stable on some classic graphs and showed that deciding the core-stability of the grand coalition is NP-hard. When looking for a coalition structure where they proposed *average payoff (AP)* algorithm to form coalitions which will be used as a another baseline algorithm in this paper.

II. Preliminaries

We represent players of a game by a set of agents $N = \{1, \ldots, n\}$. The agents may form coalitions which are represented by subsets of N. Only agents in the same *coalition* $C \subseteq N$ are accessible from each other. A *singleton coalition* contains one agent. $N_i = \{C \subseteq N \mid i \in C\}$ is denoted by the set of coalitions containing agent i. A *coalition structure* S is the set of equivalence classes of an equivalence relation defined on N. $\{N\}$ is denoted as the *grand coalition structure* and N is called *grand coalition*.

A *hedonic game* over a set of players N is characterized by a *value function* $v_i \colon N \to \mathbb{R}$ for each $i \in N$, i.e., $v_i(j)$ represents the value brought to i by the social tie with j. Naturally, a player i would prefer a coalition $C \in N_i$ that would bring a higher vale.

Definition 1. *A* fractional hedonic game (FHG) *is a hedonic game* $(N, (v_i)_{i \in N})$ *where* N *is a set of players and each* $v_i \colon N \to \mathbb{R}$ *is a value function. For any* $i \in N$ *and* $C \subseteq N$*, the* utility *of the coalition* C *to* i *is defined as*

$$v_i(C) = \begin{cases} \frac{\sum_{j \in C} v_i(j)}{|C|} & \text{if } i \in C \\ 0 & \text{otherwise} \end{cases}$$

An FHG can be represented by a graph where nodes represent agents and any edge from j to i is weighted by $v_i(j)$ between them. An FHG is *symmetric* if $v_i(j) = v_j(i)$ and *simple* if $v_i(j) \in \{0, 1\}$ for all $i, j \in N$. Thus, the graphic representation of a symmetric and simple FHG is an undirected graph $G = (V, E)$ where $N = V$ and $v_i(j) = 1$ iff $\{i, j\} \in E$. A directed and weighted graph $G = (V, E, w)$ with $w \colon E \to \mathbb{R}$ can represent an FHG which is not symmetric nor simple. The weight of its edges is represented by $v_i(j)$, the value of agent i for agent j. Assuming that the agents are connected by social relations, we view a *social network* as a graph where edges represent social relations between agents.

Given a coalition structure S, let $S(i)$ denote the coalition containing i. *Social welfare* of S is defined as $w(S) = \sum_{i \in N} v_i(S(i))$. A coalition structure is *socially optimal* if it maximizes its social welfare. We focus on the *coalition generation problem*, which takes as input a graph G that represents an FHG and seeks for a coalition of maximum social welfare.

An Agent-based Coalition Formation Framework: We adopt a natural agent-based mechanism of coalition formation (as in Alg. 1). Upon initialization, each agent forms a singleton coalition. Each agent successively computes the *payoff* that she will receive if joining another coalition in the current game. If there is a coalition from which the agent is able to obtain a higher payoff, the agent will withdraw from the current coalition and join the coalition which gives her the highest payoff. Otherwise, the agent stays within the same coalition. This finishes one *round* of the process. The process stabilizes and stops when no agent can get a higher payoff by joining another coalitions.

Benchmark: CFPO Algorithm: In the processes of coalition formation, we need to specify the amount of payoff an agent may receive when she joins a coalition. By changing the welfare distribution mechanism, a member i in a coalition C may obtain different amounts of payoff and this obviously has an impact on the decision of i. The de facto payoff of i being in the coalition C in an FHG is given by the utility $v_i(C)$. By utilizing this payoff in Alg. 1, we obtain the so-called *coalition formation with popularity-based welfare*

Algorithm 1 The Process of Coalition Formation

INPUT A graph G
OUTPUT A coalition structure S
  $S \leftarrow \{\{1\},\{2\},...\{n\}\}$ ▷ Initialize the current S
  $maxS := S$ ▷ Initialize the socially optimal S
  do
    for $i = 1$ to n **do**
      $maxP$:= current payoff of i; $maxC$:= current coalition of i
      for C in S **do**
        if $p_i(C) > maxP$ **then**
          $maxP \leftarrow p_i(C)$; $maxC \leftarrow C$
        end if
      end for
      if $maxP$ changed **then**
        i joins $maxC$ ▷ S has changed
        Recalculate payoff of agents in former and new coalitions
      end if
      if $w(S) > w(maxS)$ **then**
        $maxS := S$
      end if
    end for
  while S changed and $\text{roundCounter} \leq 1000$
  return $maxS$

distribution (CFPO). Example 2 illustrates the workings of the CFPO.

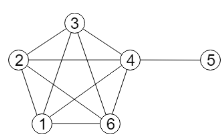


Fig. 1. The graph considered in Example 2.

Example 2. *Table I illustrates the CFPO on the graph in Fig. 1. Each row represents a round and S_t represents the coalition structure at round t. The columns represent "the agent to transfer","the coalition i joins","ith payoff","the current coalition structure after the transfer" and "social welfare", respectively. When $S_5 = \{\{1,2,3,4,5,6\}\}$, CFPO stabilizes and outputs S_5 whose social welfare is 3.67.*

TABLE I
THE PROCESSES OF COALITION FORMATION

i	Coalition	pv_i	S_t	w
			$\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}\}$	0.00
1	$\{2\}$	0.50	$\{\{1,2\},\{3\},\{4\},\{5\},\{6\}\}$	1.00
2	$\{1,2\}$	0.50	$\{\{1,2\},\{3\},\{4\},\{5\},\{6\}\}$	1.00
3	$\{1,2\}$	0.67	$\{\{1,2,3\},\{4\},\{5\},\{6\}\}$	2.00
4	$\{1,2,3\}$	0.75	$\{\{1,2,3,4\},\{5\},\{6\}\}$	3.00
5	$\{1,2,3,4\}$	0.20	$\{\{1,2,3,4,5\},\{6\}\}$	2.80
6	$\{1,2,3,4,5\}$	0.67	$\{\{1,2,3,4,5,6\}\}$	3.67

CFPO has in fact failed to find the optimal coalition structure in Example 2 as the optimal coalition structure is $\{\{1,2,3,4,6\},\{5\}\}$ with social welfare 4. The fact that player 5 joins the coalition $\{1,2,3,4\}$ reduces the overall utility of the coalition. The problem lies in *externality*, i.e., that 5 would obtain an increase in its payoff upon joining the coalition. In a fairer system, on the other hand, 5 should not obtain a positive payoff thus forbidding her to join. Based on this intuition, we propose to use an alternative payoff function. Note that this would, in principle, change the definition of the game; however, the resulting social welfare will still be consistent with FHG.

III. The CFSV Algorithm

Shapley value (SV) is a classical economic tool that distributes surplus of cooperation between players in a fair way [14]. SV is the unique value to satisfy *symmetry*, *dummy player* and *additivity*. In this paper, we adopt SV as the welfare distribution method to form coalitions instead of a solution concept. *Marginal contribution* is the surplus that an agent brought to a coalition. We denote $mc_i(C) = v(C \cup \{i\}) - v(C)$ as the marginal contribution of agent i to a coalition C. A permutation π of C can be regarded as an order in which the agents join (form) C. The marginal contribution $mc_i(\pi)$ is defined as the difference of the value of the coalition formed by the sequence ending at i and that ending at the agent in front of i. E.g., if $\pi = (1,2,3,5,4,6)$, $mc_5(\pi) = v(\{1,2,3,5\}) - v(\{1,2,3\})$. To guarantee fairness, SV takes all possible permutations Π into consideration. Π_C denotes all permutations of C.

Definition 3. *The* Shapley value $sv_i(C)$ *of i is defined as*

$$sv_i(C) = \frac{1}{|C|!} \sum_{\pi \in \Pi_C} mc_i(\pi).$$

We use SV as our welfare distribution mechanism in Alg. 1 and name the corresponding scheme *coalition formation with Shapley value-based welfare distribution (CFSV).*

Example 4. *Consider the graph in Fig. 1. Upon initialization, $S_0 = \{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}\}$. If agent 1 joins $\{2\}$, her payoff will be $sv_1(\{1,2\}) = \frac{1}{2}$. Considering all the current coalitions, agent 1 will cooperates with agent 2 to form $\{1,2\}$. Agent 2 will stay at her current coalition as no other coalition gives her higher payoff. When agent 3 considers joining $\{1,2\}$, the payoff 3 can get is $sv_3(\{1,2,3\}) = \frac{1+1+1+1+0+0}{6} = \frac{2}{3}$, which is highest among all coalitions. Agent 4 will then join $\{1,2,3\}$ for the same reason. If agent 5 joins $\{1,2,3,4\}$, the payoff will be $sv_5(\{1,2,3,4,5\}) = -0.015$. Thus 5 would stay at her current coalition. Agent 6 will join $\{1,2,3,4\}$. After comparing social welfare of all coalition structures that have been formed, CFSV outputs the optimal coalition structure $\{\{1,2,3,4,6\},\{5\}\}$.*

We next prove that CFSV may achieve optimal coalition structure for some standard graph classes [18].

A. Complete Graphs

A graph K_n is *complete* if any pair of nodes is linked by an edge.

Theorem 5. *The resultant coalition structure of CFSV on K_n is the grand coalition structure and is optimal.*

Proof. Note that a non-empty subgraph with size l is $K_l \subseteq K_n$. Knowing that $v(K_l) = l - 1$, $mc_i(K_l) = v(K_l) - v(K_{l-1}) = 1$. $mc_i = 0$ if node i is the first one in a permutation. $mc_i = 1$ in the remaining $(l! - (l-1)!)$ permutations. Thus, $sv_i(K_l) = (l! - (l-1)!)/l! = (l-1)/l$ and it increases with l.

Node 1 firstly joins $\{2\}$ and then node 2 stays put in $\{1, 2\}$. For any $3 \le t \le n$, node t faces $\mathcal{S}_{t-1} = \{K_{t-1}, \{t\}, \{t+1\}, \dots, \{n\}\}$, node t will join K_{t-1} as it gives her highest payoff. Now $\mathcal{S}_t = \{K_t, \{t+1\}, \dots, \{n\}\}$. After all the n nodes have made their turns, the resultant coalition structure is $\mathcal{S}_n = \{K_n\}$. No node would leave this grand coalition for higher payoff. CFSV outputs the grand coalition structure finally.

For optimality, define a coalition structure $S = \{C_1 \cup C_2 \cup \cdots C_k\}$ on K_n. Every induced subgraph of the complete graph is itself a complete graph, so $v(C_j) = |C_j| - 1$. Plus, $\sum_{j=1}^{k} |C_j| = n$. Social welfare of S is $w(S) = \sum_{j=1}^{k} v(C_j) = n - k$. Thus, the optimal social welfare occurs when S contains only one coalition. □

B. Paths

A *path graph* P_n is a graph whose nodes can be listed in the order $a_1, a_2, ..., a_n$ such that the edges are $\{a_i, a_{i+1}\}$ where $a = 1, 2, ..., n-1$.

Theorem 6. *On P_n, if n is even, CFSV outputs $\{\{a_1, a_2\}, \{a_3, a_4\}, \dots, \{a_{n-1}, a_n\}\}$. If n is odd, CFSV outputs $\{\{a_1, a_2\}, \{a_3, a_4\}, \dots, \{a_{n-2}, a_{n-1}, a_n\}\}$. Furthermore, the resultant coalition structure is optimal.*

Proof. $v(P_k) = 2(k-1)/k = 2 - 2/k$ on path P_k. When the path has an even number of nodes, a_1 will join $\{a_2\}$ to form $\{a_1, a_2\}$, a_2 in $\{a_1, a_2\}$ will remain. a_3 will join $\{a_4\}$ to form $\{a_3, a_4\}$, as $sv_3(P_3) = 11/72$ when she joins $\{1, 2\}$. So on so forth, other nodes form P_2 coalitions in the same way. Thus, the result is the first structure in this theorem.

When the path has an odd number of nodes, S_{n-1} on round $n-1$ consists of all P_2's except for the singleton coalition $\{a_n\}$. When a_n takes her turn, she will join the only adjacent P_2 coalition $\{a_{n-2}, a_{n-1}\}$ and forms $\{a_{n-2}, a_{n-1}, a_n\}$. we omit situations that node i joins a coalition that has no edge with i as she cannot get positive payoff.

The coalition structure in Theorem 6 is optimal which immediately follows from the proposition: For $a, b \ge 2$, $v(P_{a+b}) \le v(P_a) + v(P_b)$, which can be easily illustrated. □

C. Cycles

A *cycle* C_n is a graph whose nodes can be listed in the order $a_1, a_2, ..., a_n$ such that the edges are $\{a_i, a_{i+1}\}$ where $a = 1, 2, ..., n-1$ and $\{a_n, a_1\}$

Theorem 7. *An even cycle has the final coalition structure of $\{\{a_1, a_2\}, \{a_3, a_4\}, \dots, \{a_{n-1}, a_n\}\}$. If the cycle is odd, then the final coalition structure is $\{\{a_n, a_1, a_2\}, \{a_3, a_4\}, \dots, \{a_{n-2}, a_{n-1}\}\}$.*

We omit the proof as it is almost identical to that of Theorem 6.

Theorem 8. *The socially optimal coalition structure on C_n ($n \ge 4$) consists of all P_2's, plus a P_3 when the cycle is odd.*

Proof. First, we show that social welfare of two path-coalitions is better than that of the grand coalition. Suppose $a + b = n$ with $a, b \ge 2$. Here, P_a and P_b form a coalition structure on C_n, with social welfare of $2(a-1)/a + 2(b-1)/b$. Some elementary algebraic manipulations will show that $v(P_a) + v(P_b)$ is bigger than $v(C_n) = 2$.

Then, splitting each of these path-coalitions increases the total welfare, and thus the best way to further split the cycle up is to have as many P_2's as possible with a P_3 when the cycle is odd. □

D. Star Graphs

A n-star C_n consists of a central node c connected to $(n-1)$ disjoint tail nodes.

Theorem 9. *CFSV outputs grand coalition structure on C_n.*

Proof. Because each tail node is only connected to c, they will join and stay in whichever coalition that contains c to get a positive payoff. If c is a singleton and takes turn first, then she will join the first tail node. Or if some tail nodes have already joined $\{c\}$, then c belongs to a k-star coalition C_k with $k \ge 3$, with $(n-k)$ singleton tail nodes remaining. As the payoff of c in C_k is always bigger than $1/2$. Hence, c will stay in C_k rather than forming a P_2 with other singleton tail node.

Therefore, all the tail nodes will end up in the same coalition as c which is the grand coalition. □

Theorem 10. *The grand coalition structure is socially optimal on C_n.*

Proof. As each node is connected to c, its social welfare solely depends on the C containing c. If C has k nodes, $v(C) = 2(k-1)/k = 2 - 2/k$. $v(C)$ increases with k. Therefore, the grand coalition optimizes the social welfare. □

E. Coalition Formation with Approximate SV-based Welfare Distribution

Computing CFSV is very time consuming due to the need to calculate the marginal contribution of each agent $i \in C$ in all possible $|C|!$ permutations. Thus, it is not feasible to evaluate the performance of CFSV on large-scale graphs due to the high computational complexity. In order to evaluate the CFSV on larger graphs, we proposed an approximation algorithm CFSV_8 which computes the approximate Shapley value based on work proposed by Narayanam and Narahari [19].

Consider the case where $|C| > 8$, in this case $|C|!$ becomes very large and it becomes computationally expensive to generate all permutations to compute the Shapley value. Instead, we can use n^2 random permutations to attain the approximate Shapley value of i. Algorithm 2 shows the processes of computing the approximate Shapley value $sv_i(C)$ where the process of forming coalitions remains the same as in CFSV's.

Algorithm 2 Processes of Computing $sv_i(C)$ in CFSV_8

```
INPUT  A graph G, a coalition C, an agent i ∈ C
OUTPUT  The Shapley value of i when joining C
  if |C| > 8 then
      sum := 0
      generate n^2 permutations Π_C of C randomly
      for π in Π do
          sum := sum + mc_i(π)
      end for
      sv_i(C) := sum/n^2
  else
      calculate sv_i(C) for all possible permutations of C
  end if
  return sv_i(C)
```

IV. Experiments

We compare the resulting social welfare of the three proposed coalition formation algorithms, CFPO, CFSV and CFSV_8 against two comparative baseline algorithms on both real-world and synthetic networks.

A. Comparative Algorithms

Two comparative algorithms are used in this paper. The first uses *maximal matching* (MM) to obtain coalitions [1]. For each edge $\{i, j\}$ in a maximal matching, node i and j forms a coalition and any other unmatched nodes forms a singleton coalition separately. The authors proved that the social welfare using MM is a linear-time 4-approximation algorithm which means the resulting social welfare is at least as good as one fourth of optimal social welfare. The second algorithm is the *average payoff* (AP) algorithm [5]. Given a graph, AP firstly finds all possible coalitions formed only by a node and its neighbors, the coalition whose average payoff is the highest will be formed. Iterate using the remaining nodes not in a coalition until no coalitions can to be formed.

B. Data Sets

We evaluate algorithms on 6 real-world networks: Rhesus monkey grooming (RMG) [20], karate club (ZA) [21], Mexican political elite (MPE) [22], dolphins (DO) [23], Italian gangs (IG), and football (FB) [24]. The details of these networks are listed in Table. II where type *UU* denotes undirected unweighted graphs and *DW* denotes directed weighted graphs.

TABLE II
Parameters of Networks

Networks	Nodes	Edges	Type
RMG	16	69	DW
ZA	34	78	UU
MPE	35	117	UU
DO	62	156	UU
IG	67	114	UU
FB	115	613	UU

Aside from real-world data sets, large-scale synthetic networks are also adopted to evaluate algorithms. We use the Watts-Strogatz (WS) small-world model [25] to generate networks. In WS graphs, each node in the n-node ring topology is connected to its k nearest neighbors. Then each edge is replaced with a new edge with probability p. To simulate real world, we set n to 250-1000 and its step to 250, k to 4 and p to 0.05 and 0.25. Due to the randomness of CFSV_8, we take the average of 5 repeated experimental results.

C. Results of Experiments

Table. III shows the results of the five algorithms on real-world networks, note that the MM algorithm is not defined on weighted graphs so it has no result on the RMG network. The results show that social welfare gained by using CFSV and CFSV_8 is higher than any other method on all real-world networks. Although on most networks the difference in social welfare may not seem significant when compared against CFSV and CFSV_8, this is mainly attributed to the low number of nodes and unweighted edges in the networks. However the results on RMG show obvious advantages are on weighed graphs.

TABLE III
Social Welfare on Real-World Networks

	CFSV	CFSV_8	CFPO	MM	AP
RMG	**103.167**	**103.167**	78.700	\	84.976
ZA	**14.878**	**14.878**	14.633	10.000	13.000
MPE	**21.600**	**21.600**	19.500	16.000	20.200
DO	**33.500**	**33.500**	32.081	24.000	29.652
IG	25.337	**26.280**	24.548	20.000	22.000
FB	**91.422**	**91.422**	91.200	50.000	81.428

TABLE IV
Welfare of coalition structures on small-world data sets

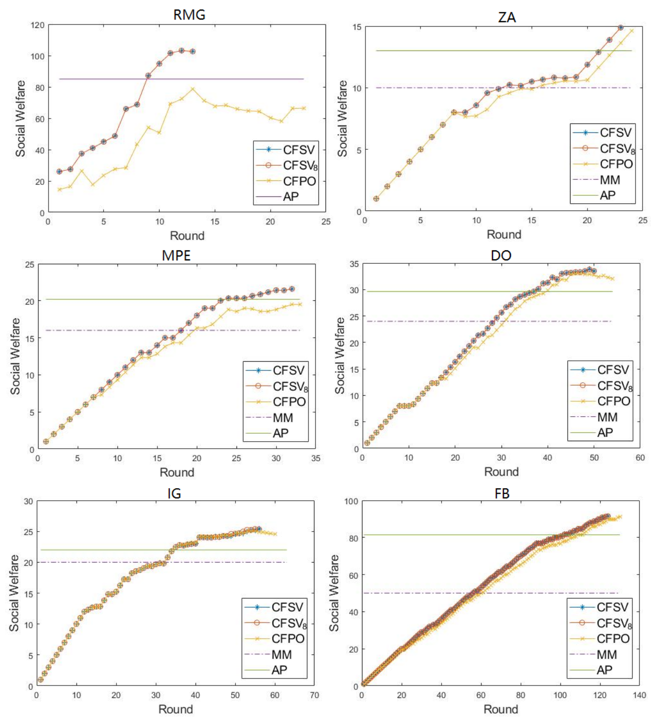
p	Nodes	$CFSV_8$	CFPO	MM	AP
0.05	250	**163.00**	157.60	124	154.93
	500	**327.00**	313.30	248	315.00
	750	**486.33**	470.10	372	466.23
	1000	**651.00**	626.80	498	631.07
0.25	250	**148.83**	146.90	121	140.4
	500	**298.67**	294.17	420	270.00
	750	**444.50**	440.37	363	410.10
	1000	**590.17**	587.50	485	548.77

As for the results shown in Table. IV, CFSV_8 has higher social welfare on all small-world graphs. Fig.2 shows the evolutionary processes of coalition formation. In each round an agent leave her current coalition to join another coalition where she gains a higher payoff. CFSV not only has higher social welfare, but also uses fewer rounds to finish forming coalitions.

V. Conclusion and Future Work

In this paper, CFPO, CFSV and CFSV_8 have been proposed to solve the coalition generation problem aiming to find a coalition structure with the maximum social welfare. Instead of

Fig. 2. The Evolutionary Processes of Coalition Formation



sticking to the de factor payoff function, we propose a framework in which the individual payoff function could be modified while preserving the coalition utility, and hence affect the coalition formation. In our paper, Shapley value has been used as a fair welfare distribution method to improve from CFPO. Not only CFSV outputs socially optimal coalition structures on complete graphs, path graphs, circles and star graphs, our experimental results on both real-world and synthetic graphs show that CFSV and CFSV$_8$ have higher social welfare in FHGs than comparative algorithms. Moreover, CFSV reduces the number of rounds of experiments. In the future, we will focus on the combinations of agents' restriction of joining a coalition and welfare distribution methods aiming at higher social welfare in FHGs. In particular, more theoretical analysis is needed to explore the effect of CFSV on the social welfare. Moreover, it will be worthwhile to study how the agent-based framework perform in dynamic social networks where the edge relation may change over time [26].

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